Forecasting Oil and Gas Using Decline Curves

Course No: P03-004
Credit: 3 PDH

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Introduction

The production of oil and gas from most wells under primary production (no injection of fluids into the reservoir) typically follows a pattern of production decline. A graph of daily or monthly production on a time scale will show a curved pattern known as a decline curve. A curve fit to this pattern allows one to forecast future production from oil and gas wells. This forecast can then be used as input to an economics model to estimate the reserves and future net income from the wells.

This course will not show methods of fitting decline curves to production data, but to use decline curves to predict future performance from oil and gas wells. The conditions necessary to use decline curves to forecast production is that wells are able to produce at unrestricted rates.

The outline of this course is as follows:

- Units of measurement in the petroleum industry
- Decline curve basics
- Exponential decline
- Hyperbolic and Harmonic decline
- Additional forecasting methods
- Hyperbolic to exponential decline
- Rate cumulative curves
- Calculation of curtailment volumes and time
- Conclusion

Units of Measurement in the Petroleum Industry

Before one can begin to discuss petroleum production, one must understand the units of measurement used in petroleum industry. Oil, gas, and natural gas liquids are the principal quantities being produced, transported, and sold in the petroleum industry. If one is to forecast future production, one must use the proper units of measurement to do so.

Oil is produced, transported, and sold in the units of barrels (abbreviated bbl or bbls). A barrel is defined as 42 US gallons and is defined at 60 ^{o}F and 14.65 pounds per square inch absolute (psia). When oil volumes are large, they may be referred to or reported in thousands or millions of barrels (Mbbl or MMbbl, respectively). Oil production rates are measured in barrels per day (bbl/d) or barrels per month (bbl/m).

Natural gas, sometimes referred to as "gas" is measured in standard cubic feet (scf) at 60 ^{o}F and various pressures in and around 14.65 psia. Each US state has their own pressure standard for the measurement of gas volumes. Since a standard cubic foot is quite small in regard to the production, transportation, and sale of natural gas, the volumes are reported in thousands of standard cubic feet (Mscf). Large volumes of gas may be referred to in millions or billions of standard cubic feet (MMscf or Bscf, respectively). Production rates of gas are measured in thousands of standard cubic feet per day (Mscfd) or millions of standard cubic feet per day (MMscfd).

Natural gas sometimes contains components that are removed by production equipment or gas plants and subsequently liquified. These are referred to as natural gas liquids (NGL). The heavier portions of the NGL are sometimes called condensate. NGLs are expressed in units of barrels (bbl). The measurements of production rates for NGL and condensate are the same as that for oil.

Below, in Table 1 are the units of measurement for volumes and flow rates of petroleum.

Table 1
Petroleum Measurement Units

Volume Units of Petroleum

Product	Units	Abbreviation
Oil	Barrels	bbl
	Thousand barrels	Mbbl
	Million barrels	MMbbl
Natural Gas	Thousand standard cubic feet	Mscf
	Million standard cubic feet	MMscf
	Billion standard cubic feet	Bscf
Condensate	Barrels	bbl
	Thousand barrels	Mbbl
	Million barrels	MMbbl
NGL	Barrels	bbl
	Thousand barrels	Mbbl
	Million barrels	MMbbl

Production Rate Units of Petroleum

Product	Units	Abbreviation
Oil	Barrels per day	bbl/d
	Barrels per month	bbl/m
Natural Gas	Thousand standard cubic feet per day	Mscfd
	Million standard cubic feet per day	Mscfm
	Thousand standard cubic feet per month	MMscfd
	Thousand standard cubic feet per month	MMscfm
Condensate	Barrels per day	bbl/d
	Barrels per month	bbl/m
NGL	Barrels per day	bbl/d
	Barrels per month	bbl/m

Decline Curve Basics

When oil and gas is initially produced it is known as primary production. In most cases, the primary production rate declines over time. There are, however, a few exceptions to this as in the case of a strong water-drive reservoir or production rates that may be limited to production facilities. Production from a strong water-drive reservoir may initially be flat. However, as oil or gas is produced, the water moves toward the well and eventually begins to be produced at which time the oil or gas production begins to decline. In cases where the production is limited by production facilities (i.e., offshore production, pipeline limits, etc.), the production is curtailed. As time passes, the reservoir pressure declines to a point when the well can no longer produce at the curtailed rate and production begins to decline.

To increase production and reserves in oil and gas wells, water or other fluids (gas, CO_2, etc.) may be injected into the reservoir. This is most common in oil reservoirs rather than gas reservoirs. To inject fluids requires additional wells be drilled, or some of the existing producing wells be converted to injection wells. The purpose of these injected fluids is to increase the reservoir pressure and push oil and gas to the production wells. If this occurs during or after primary production, this is known as secondary recovery. If some form of secondary recovery has already been done, then it is known as tertiary recovery. The oil and gas production from secondary and tertiary recovery projects will typically increase until some point in time and then eventually decline as those injected fluids, along with oil and gas is produced.

Regardless of whether a well is under primary, secondary, or tertiary production, it should be evident that production from wells will eventually decline. That being the case, this course will present information about methods used to forecast production decline. These methods are known as decline curve forecasts.

Not long after petroleum was first discovered it was observed that production rates declined over time. Upon studying production data, a mathematical relationship was found relating the decline in production rate with time. Based on the observed data, two distinct types of decline profiles were seen – exponential decline and hyperbolic decline. In exponential decline, the rate of decline (usually expressed as a percentage loss in production rate per year) is constant as time progresses. The other decline profile observed is known as hyperbolic decline. In hyperbolic decline, the decline rate is seen to decline as time progresses. The rate of production decline is related to the decline in reservoir pressure. To this day, no one is exactly sure why production out of some reservoirs decline exponentially versus hyperbolically.

Figures 1 and 2 are graphs of forecasts following exponential and hyperbolic declines over time. Figure 1 show the forecasts projected on a Cartesian scale. From figure 1, it is difficult to tell the difference in curve shape between exponential and hyperbolic decline. However, when the forecasts are plotted on a semi-log scale, the difference becomes quite apparent. The exponential decline forecast is easily recognizable as it forms a straight line on semi-log scale. Within the oil and gas industry it is customary to plot production and production forecasts on semi-log scales.

When using production to estimate decline curve parameters, it is imperative that the wells be producing at full capacity. Any curtailment of flow rate will invalidate the use of decline curves.

Figure 1

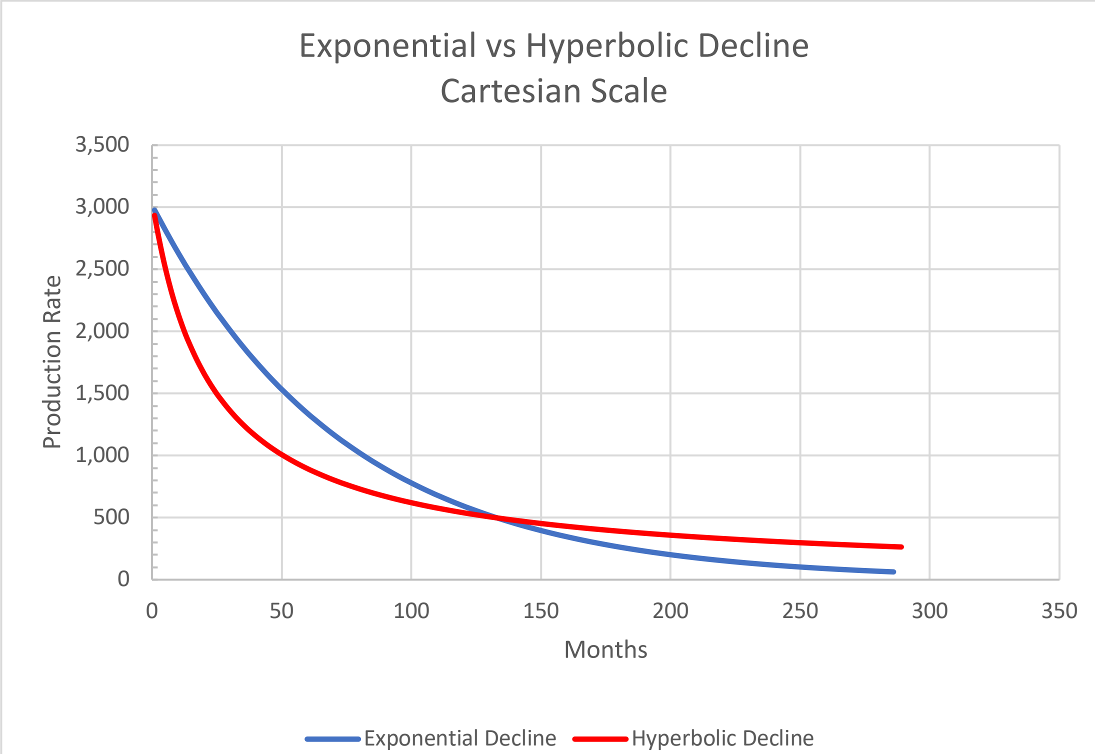
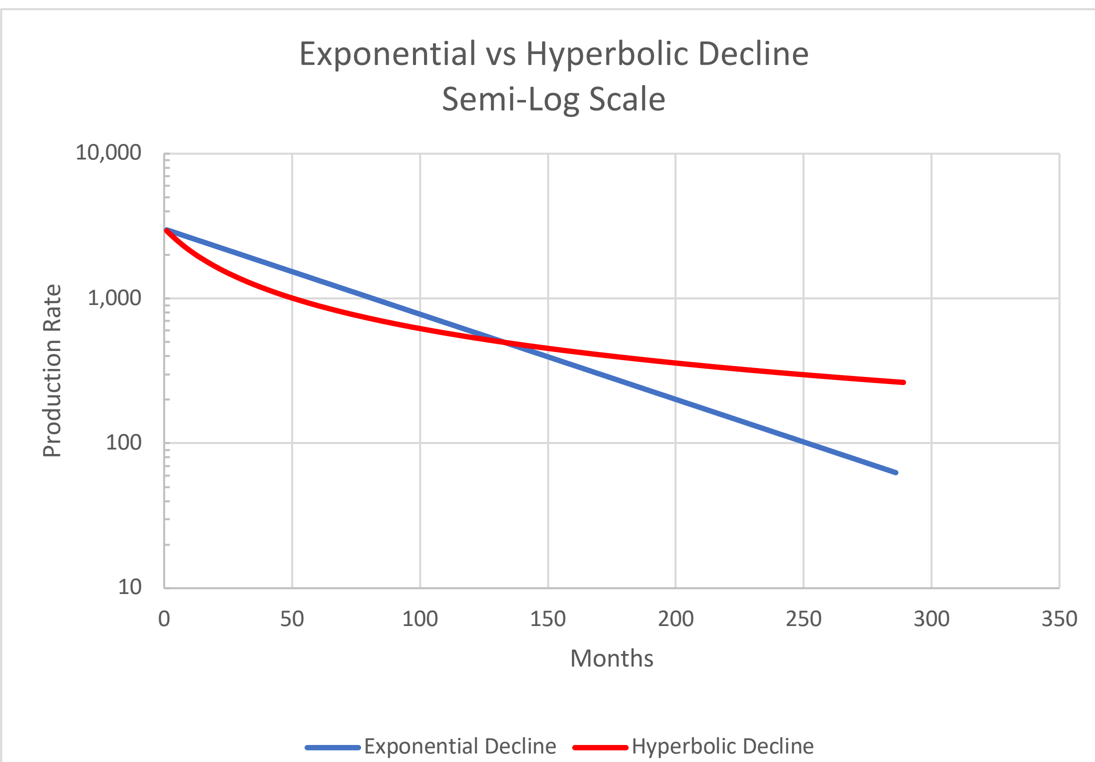


Figure 2



Exponential Decline

A production rate that declines by the same percentage each time period is known as exponential decline. If the exponential decline rate is 8% per year, it means that the production rate at the end of the year is 8% less than at the beginning of the year. The equation below shows the relationship between producing rates, time, and exponential decline rate:

$$q = q_i * (1 - d)^t$$

where,

q = producing rate at time t

q_i = initial producing rate at time t_o

d = decline rate per time period

t = time at which the calculation of q is desired

When using any of the decline equations, one must make sure that the time used in the equation matches the decline rate time frame. If one uses yearly decline rates, then the time used in the equation must be in years. If monthly decline rates are used, the time used in the equation must be months. All decline rates used in the equations are decimal, even though most times the percentage decline rate is used in discussions. The production rates, on the other hand, can be production rates over any time frame. Production rates can be daily, monthly, yearly, fortnightly, etc. Both production rates in the equation (initial and final) must be in the same units.

Example Problem:

Given an initial production rate of 120 barrels per day and an exponential decline rate of 8% per year, what is the producing rate 1 year and 2 years later?

Solution:

Since the decline rate and the time frame are both in years, one can use the exponential decline rate equation to find the production rate one year and two years hence as:

End of year 1 $q = 120 * (1 - 0.08)^1 = 110.4 \; bbl \; per \; day$

End of year 2 $q = 120 * (1 - 0.08)^2 = 101.6 \; bbl \; per \; day$

Note that the units for the production rate are in barrels per day. Notice also that the ratio of any year to the previous year is (1 – d). In this case (1 – d) = 0.92. Shown below is the relationship between production rate at the end of year 2 divided by the production rate at the end of year 1:

$$\frac{101.6}{110.4} = 0.92 = (1 - 0.08) = (1 - d)$$

It is sometime necessary to change the decline rate in one set of time units to a decline rate in another time unit. The next several equations show how to change decline rates between various sets of units.

To change decline rate from yearly to monthly:

$$d_m = 1 - (1 - d_y)^{(\frac{1}{12})}$$

where,

d_m = monthly decline rate (decimal per month)
d_y = yearly decline rate (decimal per year)

To change decline rate from monthly to yearly:

$$d_y = 1 - (1 - d_m)^{12}$$

Example Problem:

Convert a yearly decline rate of 8% to a monthly decline rate. Given an initial production rate of 120 bbl per day, use the converted decline rate to calculate the production rate after 2 years (24 months).

Solution:

Monthly decline rate $d_m = 1 - (1 - 0.08)^{\left(\frac{1}{12}\right)} = 0.006924\ per\ month$

Production rate end of year 2 $q = 120 * (1 - 0.006924)^{24} = 101.6\ bbl\ per\ day$

Notice that the production rate is identical to the solution of the previous example problem.

It is necessary to be able to calculate the oil or gas forecast between two endpoint production rates or over a certain period of time. If an initial and final production rate is known, then the following equation can be used to calculate the oil or gas production between those endpoints as:

$$N_p = \frac{(q_i - q)}{-\ln(1-d)}$$

where,

N_p = cumulative oil or gas volume
q_i = initial production rate
q = ending production rate
d = decline rate (decimal per time period)

To use this equation, the time period used in the decline rate must match the time period used in the producing rate. If the decline is decimal per year, then the production rate must be units per year. If the time units for the decline and production rate are the same, then this equation can be used as is. However, if the production rate is in daily units and the decline rate is yearly, then the following equation will properly convert units:

$$N_p = \frac{365 * (q_i - q)}{-\ln(1-d)}$$

If the production rate is in units per month and the decline is yearly, then this equation should be used:

$$N_p = \frac{12 * (q_i - q)}{-\ln(1-d)}$$

In cases where the initial production rate and decline rate are known, cumulative production at a time in the future can be calculated as:

$$N_p = \frac{q_i * (1 - (1-d)^t)}{-\ln(1-d)}$$

If the decline rate is yearly and the production rate in units per day, the production rate must be multiplied by 365 to get a correct value of N_p. Similarly, if the decline rate is yearly and the production rate in units per month, then the production rate must be multiplied by 12 to calculate N_p properly.

The following example problem will illustrate how to calculate cumulative forecast volumes.

Example Problem:

An oil well produces at an initial rate of 150 bbl per day. The production rate declines at a rate of 15% per year. What is the production rate at the end of year 3? How much oil has been produced by the end of year 3?

Solution:

The production rate at the end of year 3 is calculated as:

$$q = q_i * (1 - d)^t = 150 * (1 - 0.15)^3 = 92.1\ bbl\ per\ day$$

There are two ways one can calculate the cumulative oil. Let's calculate it both ways to check the results. Since we know the starting and ending production rates (daily rates) and the decline rate (yearly decline), the cumulative oil can be calculated as:

$$N_p = \frac{365 * (q_i - q)}{-\ln(1 - d)} = \frac{365 * (150 - 92.1)}{-\ln(1 - 0.15)} = 129{,}995\ bbl$$

Using the initial production rate, yearly decline rate, and time, we can use this equation to calculate the cumulative forecast production:

$$N_p = 365 * \frac{q_i * (1 - (1 - d)^t)}{-\ln(1 - d)} = 365 * \frac{150 * (1 - (1 - 0.15)^3)}{-\ln(1 - 0.15)} = 129{,}995\ bbl$$

Hyperbolic and Harmonic Decline

Hyperbolic decline occurs when the rate of decline decreases over time. The amount of decrease in the decline rate is constant and is defined by something termed the "b" factor. Notice in Figures 4 and 5 that there is a bend in the hyperbolic decline curve. This is especially noticeable in Figure 5 when plotted on a semi-log scale. The amount of bend in the curve is dependent on the b factor. The higher the b factor, the more the curve bends. The steepness of the hyperbolic decline curve is determined by the initial decline rate of the production (or forecast). If the b factor is zero, the rate decline is exponential.

The b factor and initial decline rate are determined by fitting a hyperbolic curve to production data. There are a number of petroleum economic software programs available that can be used to fit curves to the production data. It can also be done manually, but is rather time consuming.

It was initially thought that the value for the b factor could not exceed the value of 1.0 as most reservoirs had b factors less than 1.0. However, with the advent of hydraulic fracturing and production from shale, b factors as high as 2.0 have been observed. When the b factor is equal to 1.0, it is known as harmonic decline. When discussing hyperbolic decline, it is intended to also include harmonic decline.

To forecast production rates at future times following hyperbolic decline, the following equation ca be used:

$$q = \frac{q_i}{(1 + b * d_i * t)^{\frac{1}{b}}}$$

where,

q = production rate at time t
q_i = initial production rate
b = b factor
d_i = initial hyperbolic decline rate (not the initial effective decline rate – see below)
t = time

The decline rate is a hyperbolic decline rate and must be calculated from the effective decline rate. The effective decline rate is the decline that would occur if it were exponential. To calculate the hyperbolic decline rate use:

$$d_i = -\ln{(1 - d_e)}$$

where d_e is the effective decline rate.

If the b factor is not equal to 1.0, the following equation will calculate the cumulative volume between an initial and final production rate:

$$N_p = \frac{{q_i}^b}{d_i * (1 - b)} * (q_i^{(1-b)} - q^{(1-b)})$$

Using the initial rate, initial hyperbolic decline rate, and b factor not equal to 1.0, the cumulative production to a future time can be calculated as:

$$N_p = \frac{q_i}{d_i * (1 - b)} * (1 - (1 + b * d_i * t)^{\left(\frac{b-1}{b}\right)})$$

Remember that the units must be consistent. If using daily rates and yearly declines, the rate must be multiplied by 365. If monthly rates are used with yearly declines, the rate must be multiplied by 12.

For the case of harmonic decline when the b factor is equal to 1.0, the equations are much simpler. The following equations can be used to calculate cumulative production between two production rates or at some future point in time:

Based on initial an ending production rates:

$$N_p = \frac{q_i}{d_i} * \ln\left(\frac{q_i}{q}\right)$$

When using the initial rate, initial hyperbolic decline, and time, use:

$$N_p = \frac{q_i}{d_i} * \ln(1 + d_i * t)$$

The following example problem will be used illustrate the use of hyperbolic decline equations in calculating future rates and cumulative production volumes. For this example, we will use gas production.

Example Problem:

A shale gas well initially produces at a rate of 2,500 Mscfd (thousand standard cubic feet per day). The reservoir is known to produce hyperbolically with a b factor of 1.20. The effective initial decline rate has been calculated to be 65% per year.

What is the producing rate and cumulative gas production at the end of year 2?

Solution:

The first thing to calculate is the hyperbolic decline rate as:

$$d_i = -\ln(1 - d_e) = -\ln(1 - 0.65) = 1.0498$$

The rate at the end of year 2 is calculated as:

$$q = \frac{q_i}{(1 + b * d_i * t)^{\frac{1}{b}}} = \frac{2{,}500}{(1 + 1.2 * 1.0498 * 2)^{\frac{1}{1.2}}} = 876\ Mscfd$$

The cumulative volume can be calculated from the initial and ending rate as:

$$N_p = 365 * \frac{2{,}500^{1.2}}{1.0498 * (1 - 1.2)} * \left(2{,}500^{(1-1.2)} - 876^{(1-1.2)}\right) = 1{,}014{,}165\ Mscf$$

The cumulative volume can also be calculated as:

$$N_p = 365 * \frac{2{,}500}{1.0498 * (1 - 1.2)} * \left(1 - (1 + 1.2 * 1.0498 * 2)^{\left(\frac{1.2-1}{1.2}\right)}\right) = 1{,}014{,}083\ Mscf$$

The difference between the two volumes is round-off error.

Table 2 lists equations used to calculate future rates and volumes for exponential, hyperbolic, and harmonic declines.

Table 2
Properties of Decline Types

Decline Type	Rate Time	Cumulative Volume
Exponential	$q = q_i * (1 - d)^t$	$N_p = \frac{(q_i - q)}{-\ln(1-d)}$ or $N_p = \frac{q_i * (1 - (1-d)^t)}{-\ln(1-d)}$
Hyperbolic	$q = \frac{q_i}{(1 + b * d_i * t)^{\frac{1}{b}}}$	$N_p = \frac{{q_i}^b}{d_i * (1-b)} * (q_i^{(1-b)} - q^{(1-b)})$ or $N_p = \frac{q_i}{d_i * (1-b)} * (1 - (1 + b * d_i * t)^{\left(\frac{b-1}{b}\right)})$
Harmonic	$q = \frac{q_i}{(1 + d_i * t)}$	$N_p = \frac{q_i}{d_i} * \ln\left(\frac{q_i}{q}\right)$ or $N_p = \frac{q_i}{d_i} * \ln(1 + d_i * t)$

Additional Forecasting Methods

Although it is perfectly suitable to forecast production using decline curves, secondary products (gas in the case of an oil well, condensate and NGL in the case of gas wells) can also be calculated using ratios to the primary product.

For oil wells where the gas production closely follows the oil production, one can use the ratio of gas to oil to forecast the gas production. The ratio of gas to oil is known as the gas-oil-ratio, or GOR and is expressed in units of standard cubic feet per barrel (scf/bbl). For reporting and economic calculations, it proper to calculate the gas rates in Mscf. To calculate the gas production, given the oil production and GOR, use the following formula:

$$q_g = \frac{q_o * GOR}{1{,}000}$$

where,

q_g = gas production (Mscfd or Mscfm)
q_o = oil production (bbl/d or bbl/m)
GOR = gas-oil-ratio (scf/bbl)

For gas production, we can have associated condensate and NGL production. Although the ratio of these products to gas production can change over time, their production can be forecast as ratios to gas production. The ratios of condensate production to gas production and NGL production to gas production are called Condensate Yield, and NGL Yield, respectively. When using yields, one must use the wellhead gas volumes and not sales gas volumes. The units for condensate and NGL yields are barrels per million standard cubic feet (bbl/MMscf). The following equations can be used to calculate condensate and NGL production as ratios of gas production:

$$q_c = q_g * \frac{Yield_c}{1{,}000} \qquad q_{NGL} = q_g * \frac{Yield_{NGL}}{1{,}000}$$

where,

q_c = condensate production (bbl/d or bbl/m)
q_{NGL} = NGL production (bbl/d or bbl/m)
q_g = wellhead gas production (Mscfd or Mscfm)
$Yield_c$ = condensate yield (bbl/MMscf)
$Yield_{NGL}$ = NGL yield (bbl/MMscf)

Hyperbolic to Exponential Decline

It is established that production which initially declines hyperbolically will eventually decline exponentially. The decline rate at which this happens is known as the "minimum decline rate" or "terminal decline rate". This minimum decline rate is empirically chosen based on wells producing from the same or similar reservoirs that have already experienced the change from hyperbolic to exponential decline.

To forecast production, it is necessary to calculate the time and production rate when the decline changes from hyperbolic to exponential. If the minimum decline rate is chosen, then the time when the decline changes from hyperbolic to exponential can be calculated as:

$$t = \frac{(1 - d_m)^b * (1 + d_i * b) - 1}{d_i * b * (1 - (1 - d_m)^b)}$$

$$d_i = -\ln(1 - d_e)$$

where,

t = time when decline changes from hyperbolic to exponential (months)
d_i = initial hyperbolic decline rate (monthly decimal)
b = b factor
d_m = minimum exponential decline rate (monthly decimal)
d_e = initial effective decline rate (monthly decimal)

Using monthly decline rates and volumes allows one to solve for time in months. Not much accuracy will be lost by rounding the time to the nearest month. The following example will help show the necessary calculations.

Example Problem:

A well is declining hyperbolically with the following decline parameters:

Initial effective decline rate = 48% per year
b = 1.15
Initial production rate = 150 bbl/d

If the minimum decline rate is 8% per year, when does the decline change from hyperbolic to exponential? What is the production rate at the change? How much oil is produced under hyperbolic decline? If the final rate of production is 5 bbl/d, how much oil is produced under exponential decline?

Solution:

Calculate d_e in monthly rate as: $d_m = 1 - (1 - 0.48)^{\left(\frac{1}{12}\right)} = 0.0530$

Calculate d_i as: $d_i = -\ln(1 - 0.0530) = 0.0545$

Calculate d_m in monthly rate as: $d_m = 1 - (1 - 0.08)^{\left(\frac{1}{12}\right)} = 0.006924$

Calculate time, in months for hyperbolic to exponential decline:

$$t = \frac{(1 - 0.006924\ )^{1.15} * (1 + 0.0545 * 1.15) - 1}{0.0545 * 1.15 * (1 - \ (1 - 0.006924)^{1.15})} = 109\ months\ begin \exp decl$$

Calculate the initial production rate as monthly rate:

$$q_o = 150 * \frac{365}{12} = 4{,}563\ bbl/m$$

Calculate hyperbolic cumulative oil at 109 months:

$$N_p = \frac{4563}{0.0545 * (1 - 1.15)} * \left(1 - (1 + 1.15 * 0.0545 * 109)^{\left(\frac{1.15-1}{1.15}\right)}\right) = 171{,}883\ bbl$$

Calculate the monthly production rate at 109 months:

$$q = \frac{4563}{(1 + 1.15 * 0.0545 * 109)^{\frac{1}{1.15}}} = 762\ bbl/m$$

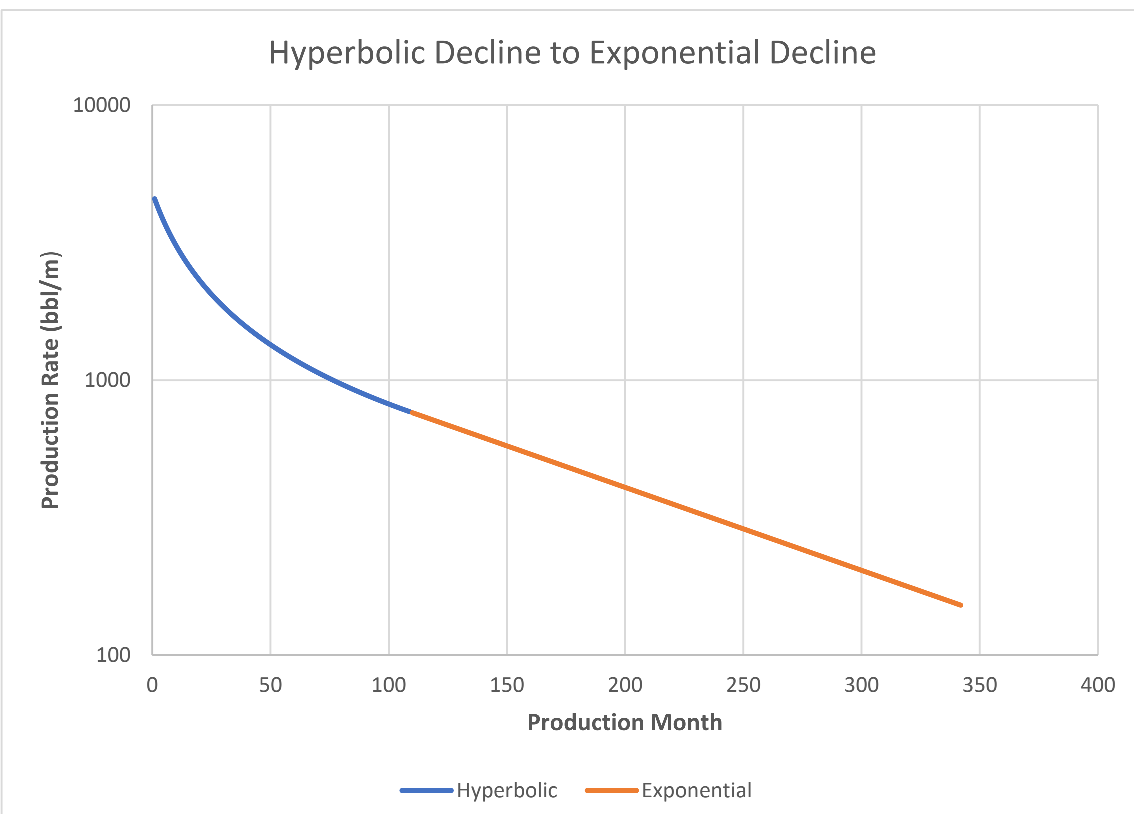
Calculate exponential ending rate at 5 bbl/d:

$$q_o = 5 * \frac{365}{12} = 152\ bbl/m$$

Calculate exponential cumulative oil at ending rate of 152 bbl/m:

$$N_p = \frac{(762 - 152)}{-\ln(1 - 0.006924)} = 87{,}789\ bbl$$

The graph below shows the decline curve defined above:



Rate Cumulative Curves

In most cases, one is interested in knowing the forecast production rate and cumulative production at a certain point in time. This is because factors such as economic outcomes and investment plans concerning oil and gas wells operate on a time scale. However, knowing the rate-time relationship, one can also create a rate-cumulative production relationship.

Rate-time decline curves are only valid for wells producing at unrestricted rates. On the other hand, the rate-cumulative relationship is independent of time. At times when a well may be curtailed or shut-in and the rate-time decline curve cannot be used, the ensuing production rate can be calculated using the rate-cumulative production curve.

To create the rate-cumulative production relationship, we simply use the unrestricted rate-time and cumulative time relationships to calculate values of rate and cumulative production at selected intervals of time. With the use of a spreadsheet program, we can calculate those values monthly. Using the equations from Table 2, a rate-cumulative relationship can be created for a well declining exponentially using:

$$q = q_i * (1 - d)^t$$

$$N_p = \frac{q_i * (1 - (1 - d)^t)}{-\ln(1 - d)}$$

Wells declining hyperbolically would use:

$$q = \frac{q_i}{(1 + b * d_i * t)^{\frac{1}{b}}}$$

$$N_p = \frac{q_i}{d_i * (1 - b)} * (1 - (1 + b * d_i * t)^{\left(\frac{b-1}{b}\right)})$$

And wells declining harmonically would use these equations:

$$q = \frac{q_i}{(1 + d_i * t)}$$

$$N_p = \frac{q_i}{d_i} * \ln{(1 + d_i * t)}$$

The following example problem will demonstrate how to create a rate-cumulative production relationship.

Example Problem:

A well is declining harmonically. The initial production rate was 120 bbl/d at an effective decline rate is 35% per year. The final production rate is estimated to be 10 bbl/d. Create a rate-cumulative curve for the first 5 years using semi-annual time frames.

Solution:

The problem can be solved using either daily or monthly rates. Since the rate-time equation uses ratios of production rates, the answer will be identical as long as the rates are in the same units. In the cumulative-time equation, the rate must be multiplied by 365 when using daily rates and 12 when using monthly rates. Since this is an example problem, we will show solutions for both daily and monthly rates and plots for both.

The first thing to do is calculate the hyperbolic decline as:

Calculate yearly d_i as: $d_i = -\ln(1 - 0.35) = 0.4308$

Monthly effective decline rate is: $d_m = 1 - (1 - 0.35)^{\left(\frac{1}{12}\right)} = 0.0353$

Calculate monthly d_i as: $d_i = -\ln(1 - 0.0353) = 0.0359$

Initial monthly production rate is:

$$q_o = 120 * \frac{365}{12} = 3{,}650\ bbl/m$$

If we use daily rates the rate at a cumulative production of zero is 120 bbl/d. The calculation of the rate and cumulative production at the first semi-annual period is:

$$q = \frac{120}{(1 + 0.4308 * 0.5)} = 99\ bbl/d$$

$$N_p = \frac{365 * 120}{0.4308} * \ln(1 + 0.4308 * 0.5) = 19{,}833\ bbl$$

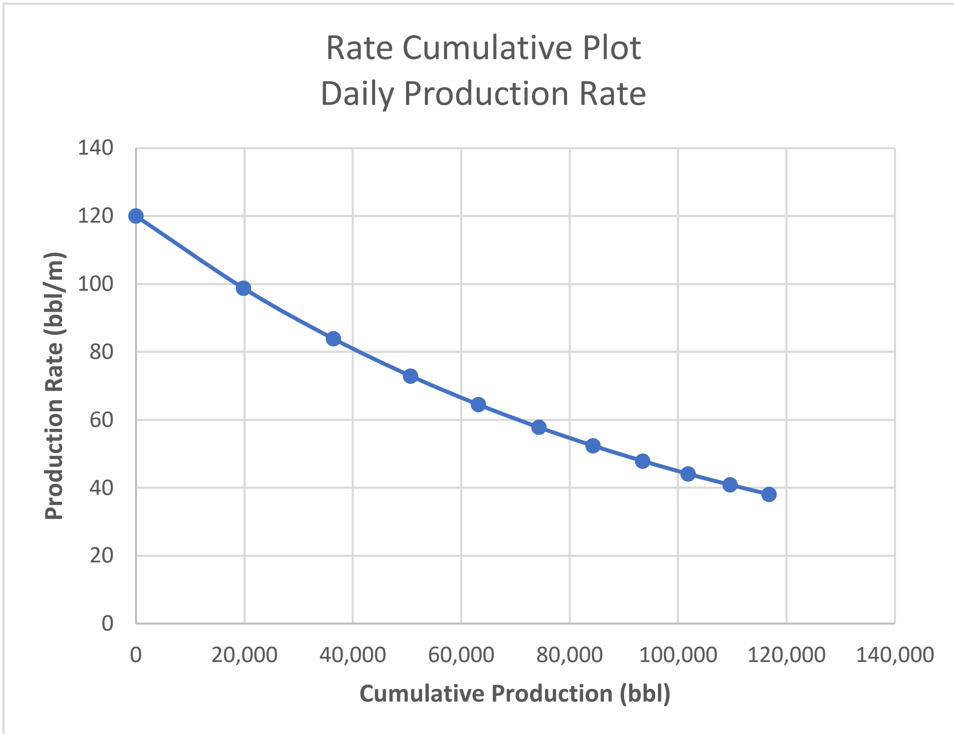
The remaining points are calculated in the same fashion. The table below shows the results for the first 5 years using daily rates.

Time (yrs)	Qo (bbl/d)	Np (bbl)
0.0	120	0
0.5	99	19,833
1.0	84	36,422
1.5	73	50,680
2.0	64	63,183
2.5	58	74,315
3.0	52	84,348
3.5	48	93,478
4.0	44	101,857
4.5	41	109,597
5.0	38	116,789

Here are the calculations monthly:

Time (mos)	Qo (bbl/m)	Np (bbl)
0	3,650	0
6	3,003	19,833
12	2,551	36,422
18	2,217	50,680
24	1,961	63,183
30	1,757	74,315
36	1,592	84,348
42	1,455	93,478
48	1,340	101,857
54	1,242	109,597
60	1,157	116,789

Shown below are the rate-cumulative production plots in daily and monthly rates:



Curtailed Production

Under certain conditions, production from oil and gas wells may be curtailed. This usually occurs when there are production limitations in the surface equipment. Examples of curtailment are offshore facilities with limited space, gathering system limits, gas plant limits, etc. Because production is curtailed, decline curves cannot be used to forecast the production while it is curtailed.

Based on an unrestricted flow rate and decline trend, one can calculate the time and production volume under curtailed conditions. One cannot just use a rate-time equation to calculate the time when the production rate will reach the curtailed rate. This is because the rate calculated using the rate-time equations give the time when an unrestricted well reaches that rate. To calculate the time when the curtailment ends, one must first calculate the cumulative production at the curtailed rate. The time of the curtailment is then found by dividing the cumulative production by the curtailment production rate. If the production trend was initially exponential, the cumulative production at the end of curtailment and the time of curtailment is found by:

$$N_{pc} = \frac{(q_i - q_c)}{-\ln(1-d)}$$

$$t_c = \frac{N_{pc}}{q_c}$$

where,

N_{pc}= cumulative volume produced under curtailment (bbl or Mscf)
q_c = curtailed production rate (bbl/m or Mscfm)
q_i = initial production rate (bbl/m or Mscfm)
d = monthly decline rate (decimal)
t_c = time of curtailment (months)

If the initial decline trend is hyperbolic then these equations can be used to solve for curtailment volume and time:

$$N_{pc} = \frac{{q_i}^b}{d_i * (1-b)} * (q_i^{(1-b)} - q_c^{(1-b)})$$

$$t_c = \frac{N_{pc}}{q_c}$$

where,

N_{pc}= cumulative volume produced under curtailment (bbl or Mscf)
q_c = curtailed production rate (bbl/m or Mscfm)
q_i = initial production rate (bbl/m or Mscfm)
d_i = initial hyperbolic monthly decline rate (decimal)
b = b factor
t_c = time of curtailment (months)

When decline is harmonic, use these equations:

$$N_{pc} = \frac{q_i}{d_i} * \ln\left(\frac{q_i}{q_c}\right)$$

$$t_c = \frac{N_{pc}}{q_c}$$

where,

N_{pc}= cumulative volume produced under curtailment (bbl or Mscf)
q_c = curtailed production rate (bbl/m or Mscfm)
q_i = initial production rate (bbl/m or Mscfm)
d_i = initial hyperbolic monthly decline rate (decimal)
t_c = time of curtailment (months)

The following example problem will provide instruction on solving a curtailment situation.

Example Problem:

A well initially produces at 1,800 Mscfd with a 12% exponential decline rate. Due to gas plant constraints, the well is curtailed to 1,200 Mscfd. When is the well expected to be back on decline? What is the amount of gas produced by the well while curtailed?

Solution:

The first thing to do is put production rates in monthly units:

$$q_i = 1{,}800 * \frac{365}{12} = 54{,}750\ Mcfm$$

$$q_c = 1{,}200 * \frac{365}{12} = 36{,}500\ Mcfm$$

Calculate monthly decline rate:

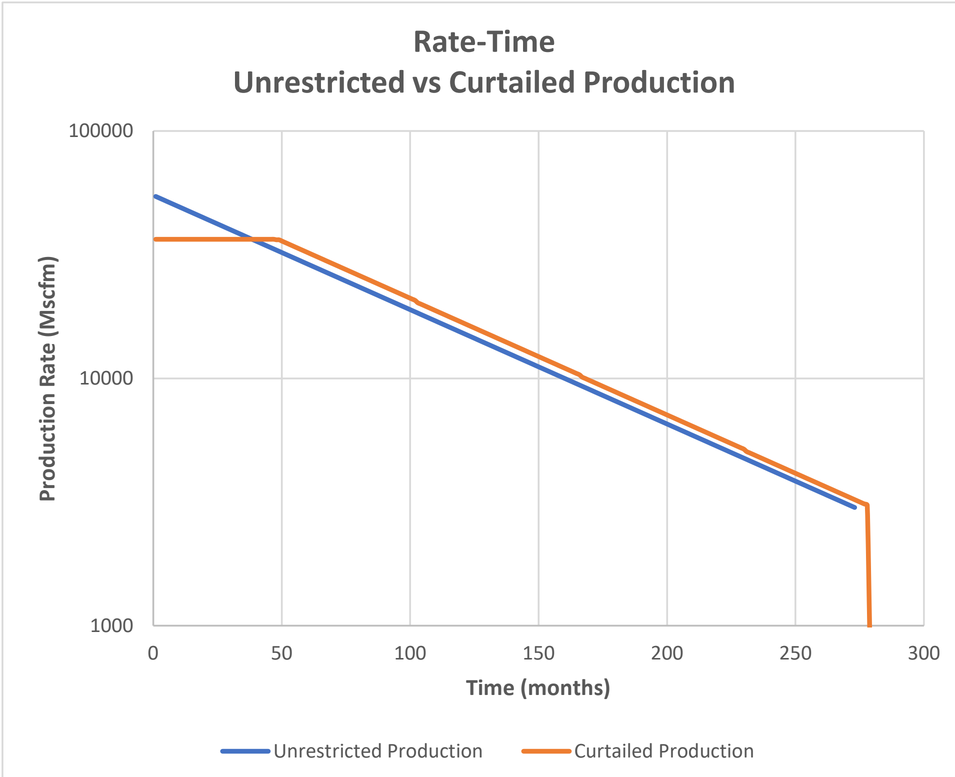
$$d_m = 1 - (1 - 0.12)^{\left(\frac{1}{12}\right)} = 0.0106$$

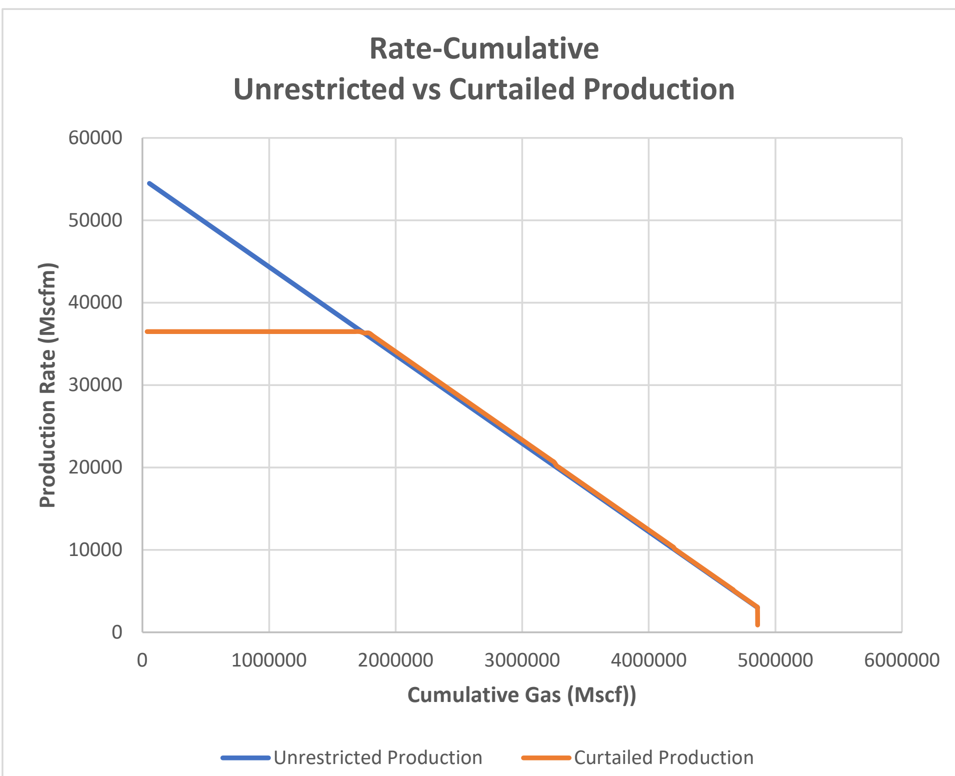
Calculate the cumulative volume and time on curtailment:

$$N_{pc} = \frac{(54750 - 36500)}{-\ln{(1 - 0.0106)}} = 1{,}712{,}557\ Mcf$$

$$t_c = \frac{1{,}712{,}557}{36{,}500} = 47\ months$$

The following graphs show the effect of curtailment on the rate-time and rate-cumulative curves.





Conclusion

One thing I hope the reader has noticed in this course is the fact that there are a lot of choices for time units when using decline curves and formulas. This can create a lot of confusion when making calculations and create doubt in the results.

The reader should choose a consistent set of units with which they are comfortable. My recommendation is to use monthly time units and calculate everything based on monthly time frames. This is also consistent with accounting practices in the oil and gas industry, as revenue, expenses, production taxes, and royalties are paid on a monthly basis. When estimating future performance of an oil or gas well, the results of decline curve calculations will be fed directly into economic software or accounting systems. When using monthly calculations, there will be no need to convert results of decline curve calculations to another system of time units.